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Fuzzy interaction modelling for participants in innovation development: approaches and examples

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Abstract: *The article considers the interaction problems of the participants in innovative development at the regional level. Mathematical approaches and formulations for modelling, such as the interaction on the basis of game approaches and the theory of fuzzy sets, have been proposed. In particular, the interaction model of innovative participants in the region, considered as a fuzzy coalition game, is presented. Its theoretical justification and an example of practical calculations are given. Further, the methodology of interaction modelling is considered, taking into account the motives of the participants in innovative development when forming fuzzy coalitions. An example of the corresponding calculations is also given. Also, the interaction model of "state-regions" in the interpretation of the fuzzy hierarchical game is proposed and described. The features of its solution are described and an example of calculations is presented.*

Key-words: *innovative development, fuzzy sets applying, interaction modelling, fuzzy games*

1. Introduction

Issues of innovative development are extremely important for the market transformation and modernization in post-Soviet and post-communist economies, in particular, such as Russia and Ukraine (Kolosok and Trusova, 2015; Untura, 2015; Kovalenko, 2015; Dnishev et al, 2015; Bezkorovainyi and Jarzębowski, 2016; Shatkovskaya et al, 2017).

At the same time, it is necessary to take into account the significant role of the state and local administrations, the presence of other various external and internal participants in this process (Przygoda, 2015; Iurchenko, 2015; Fundeanu, 2015; Simba, 2015; Makarov et al, 2016; Yordanova and Blagoev, 2016).

In such conditions, mathematically adequate modelling of all subjects of innovative development of regions becomes an important and urgent task. It is extremely necessary to use modern, adequate methods of mathematical modelling,

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appropriate models and decision-making methods based on sufficiently reasoned calculations (Gureev, 2015; Golova et al, 2017).

At the same time, despite the definite availability of scientific literature on specific issues of mathematical and economic modelling of certain aspects of innovative development at the state or regional level, the development of new approaches to resolving these issues remains highly relevant (Pashkus, 2016).

This is especially true for the interaction of participants in the investment development, which in practice occurs in the context of significant uncertainties of different origins (Lipieta and Malawski, 2016; Maltseva, 2016; Maltsevaet al, 2016).

As it is already known, for the innovative development of the region, one of the most important conditions is the favourable innovative environment. It is, among other things, determined by the narrow, transparent, and flexible relationships between key participants in the innovative activity.

Consequently, the regional innovation system is characterised by synergies, because the maximum effect can be achieved only in the case of maximizing the effectiveness of the collaborative operation of its elements. At this level, the innovation development of the region's economy depends not so much on the effective functioning of the individual economic agents, but on the interaction between all subjects in the innovative activity.

2. The interaction model of innovative participants in the region as a fuzzy coalition game

It is obvious that participants in the innovation process may have an insufficiently clear understanding of the expected results of cooperation, manifested in the form of an open, dynamic system. In doing so, they can form vague coalitions which are justified also by the fact that they make it possible for various innovative players to participate in different coalitions.

Proceeding from the facts above, the cooperation of the innovative participants at various levels of the region can be represented by fuzzy cooperative games, where players will consider partnership as a function of usefulness, which enables them to achieve the maximum individual effect (Lekeas and Stamatopoulos, 2016).

Suppose that for the implementation of the innovative program IP, consisting of several projects $P = (P_l, l = \overline{1, L})$, participants form coalitions.

Then we have a fuzzy cooperative game, which is defined by a pair $(I, \widetilde{v})$, where $\widetilde{v}$ — characteristic function of a cooperative game (satisfaction function), $I = \{1,2...m\}$ the finite and non-empty set of innovative players of the region (scientific and research centres, universities, companies, government agencies etc.), which consists of fuzzy coalitions $K \subseteq I$.

Because $\tilde{v}: 2^m \to \Re^+$ is a reflection that links $K \subseteq I$ fuzzy coalition with the winning value $\tilde{v}(K)$, then fuzzy coalition K on the set of players I is determined by the membership function $\mu_K : R \to [0,1]$, $K \neq 0$, $Sup\mu_K(\tilde{v}(K)) = 1$,
where $\tilde{v}(K)$ – the fuzzy value of winning of the coalition K.

For each innovation player i there is a function of the individual usefulness, which depends on the set of "Resources" R_g^i (it may be money, knowledge, time etc.), which they have.

Then each player i can estimate the total winnings or budget of the coalition K_j, besides K_j may take place at the budget $b_j^{\min} \le b \le b_j^{\max}$. In this case, the regional authorities can act as a mediator in the formation of the coalition, propose the optimal structure of coalition $K^* = (K_1, K_2 ... K_n)$ and the corresponding vector of the budget:

$$\tilde{v}(K)^* = \sum_{i=1}^{m} \tilde{v}(i), \text{ at } \sum_{j=1}^{n} b_j^{\min} \le \sum_{i=1}^{m} R_j^i \le \sum_{j=1}^{n} b_j^{\max}, n \le m \tag{1}$$

So, if the usefulness of the coalition is $\tilde{v}(K)$ and the division or payments vector is $s = (s_1 + s_2 + ... + s_i)$, that decision of coalition game (division) must satisfy the budget constraint, i.e. the following inequality holds:

$$s = (s_1 + s_2 + ... + s_i) \le \tilde{v}(K) \text{ or } \sum_{i \in K} s_i \le \tilde{v}(K) \tag{2}$$

For each player $i \in I$ $s_i \ge \tilde{v}(i)$, i.e. the individual rationality is not less than the coalition usefulness, because nonintersecting coalitions join together, in order to earn together not less than separately.

Thus, for vector of division s, it is necessary that

$$s_i = \tilde{v}(i) + \alpha_i, i \in K, \alpha_i \ge 0, \sum_{i \in K} \alpha_i = \tilde{v}(K) - \sum_{i \in K} \tilde{v}(i), \tag{3}$$

where α_i - additional winnings of player i from participating in the coalition.

Since in the fuzzy cooperative game each player i can participate in different coalitions $K_1, K_2 ... K_n$, then the degree of his/ her participation will be

$\sum_{i\in K}\mu_{K_n}(i)=1, \mu_{K_n}(i)\geq 0$. If $v_{K_j}(i)$ – it is winnings of player i in the coalition K_j, then his/ her total winnings depending on his/ her degree of participation in the coalition will be: $\sum_{j=1}^{n}\tilde{v}_{K_j}(i)=\dfrac{\sum_{j=1}^{n}\mu_{K_j}(i)*\tilde{v}(K_j)}{\sum\mu_{K_j}(i)}$, where $\mu_{Kj}(i)\in[0,1]$ –degree of participation of the player i in the fuzzy coalition K_j.

Hence, a final winning that coalition K guarantees to its members, there is $\sum_{K\in I}\mu_K\tilde{v}(K)$, while the maximum:

$$v^*(K)=\sup\left\{\sum_{K\in I}\mu_K\tilde{v}(K)/\sum_{j=1}^{n}b_j^{\min}\leq\sum_{i=1}^{m}R_j^i\leq\sum_{j=1}^{n}b_j^{\max}, n\leq m, \mu_K\geq 0, \sum_{i\in K}\mu_K(i)=K\right\} \quad (4)$$

To redistribute the largest fuzzy coalition winning for the coalition K, can be applied the fuzzy Shapley value (Butnariu, 1980; Yu Hsien Liao, 2013). It is a certain scheme for solving a fuzzy cooperative game, which takes into account the contribution of each player in the win:

$$\varphi_{Sh}^{i}(\tilde{v})=\sum_{i\in K}\eta(|K|)\times(\tilde{v}(K)-\tilde{v}(K/i)), \quad (5)$$

where: $\eta(|K|)=\dfrac{(m-k-1)!k!}{m!}$; $(\tilde{v}(K)-\tilde{v}(K/i))$ - additional profits which can be provided by i-th player himself/ herself.

The equitable distribution of profits is one of the important points for the effective and stable cooperation between innovative players (actors) in the region.

3. Numerical example for fuzzy coalition game model

For example, we assume that there are a lot of innovative players $I=\{I_i, i=\overline{1,5}\}$, which can form coalitions $K_j, j=\overline{1,7}$.

Suppose that, despite the necessary cooperation in carrying out a particular project, the importance (competence) of all players is different, i.e. without the full participation of players $\{I_1, I_2\}$ the coalitions winning $v(K_j) = 0$, also $v(\{I_1, I_2\}) = 0$, if others do not participate, i.e. for all j and $3 \le i \le 5$.

For simplicity, we assume that players can fully engage (1) or not be involved (0) in the coalition. Then it is obvious that:

$$K_1 = (1,\ 1,\ 1,\ 0,\ 0); K_2 = (1,\ 1,\ 0,\ 0,\ 1); K_3 = (1,\ 1,\ 0,\ 1,\ 0); K_4 = (1,\ 1,\ 1,\ 1,\ 0);$$
$$K_5 = (1,\ 1,\ 1,\ 0,\ 1); K_6 = (1,\ 1,\ 0,\ 1,\ 1); K_7 = (1,\ 1,\ 1,\ 1,\ 1);$$

Because each participant may sell their resources at a specified price and each of them can receive not less than and no more than a fixed sum, then the winning of the player can be set by triangular fuzzy number.

Having designated a fuzzy value of resources, at the disposal of players I_i and which can be transferred to other players of the coalition K, through $R_i^{K_j} = (r_i^L, r_i^*, r_i^R)$ – fuzzy *LR* – number, we can build a table of fuzzy allocation of resources (Table 1).

$R_i^{K_j}$	I_1	I_2	I_3	I_4	I_5
R_1	(1, 1.5, 2)	(4, 5, 6)	(2, 2.5,3)	(1,2,3)	(5,6,7)
R_2	(3, 3.5, 4)	(6, 6.5, 7)	(2, 3, 5)	(3,4,5)	(4.5, 5, 5.5)
R_3	(2, 2.5, 3)	(1, 1.5, 2)	(5, 5.5, 6)	(6,6.5, 7)	(1.5, 2, 2.5)
R_4	(1, 2, 2.5)	(0.5,1, 1.5)	(3,3.5, 4)	(6, 6.5, 7)	(4, 5, 5.5)
R_5	(4,5,7)	(2.5, 3, 4.5)	(7, 7.5, 8)	(2, 3, 4)	(3, 4, 4.5)
$\sum\oplus$	(11, 14.5, 21)	(14, 17,21)	(19, 22, 26)	(18, 21, 26)	(18,22, 25)

Table 1. *Allocation of resources*
(source: own authors' numerical example)

The resource budget of coalitions will be defined as follows:

coalition 1: $(11 \le R_1 \le 21, 14 \le R_2 \le 21, 19 \le R_3 \le 26)$;

coalition 2: $(11 \le R_1 \le 21, 14 \le R_2 \le 21, 18 \le R_5 \le 25)$;

coalition 3: $(11 \le R_1 \le 21, 14 \le R_2 \le 21, 18 \le R_4 \le 26)$;

coalition 4: $(11 \le R_1 \le 21, 14 \le R_2 \le 21, 19 \le R_3 \le 26, 18 \le R_4 \le 26)$;

coalition 5: $(11 \le R_1 \le 21, 14 \le R_2 \le 21, 19 \le R_3 \le 26, 18 \le R_5 \le 25)$;

coalition 6: $(11 \le R_1 \le 21, 14 \le R_2 \le 21, 18 \le R_4 \le 26, 18 \le R_5 \le 25)$;

coalition 7: $(11 \le R_1 \le 21, 14 \le R_2 \le 21, 19 \le R_3 \le 26, 18 \le R_4 \le 26, 18 \le R_5 \le 25)$.

Acting as a mediator between innovative players, regional authorities may recommend participants as the optimal coalition in the innovation process, with the corresponding budget, the one for which $\sum_{K \in I} \mu_K v(K)$ is maximal:

$$v(\{I_1\}) = v(\{I_2\}) = v(\{I_3\}) = v(\{I_4\}) = v(\{I_5\}) = 0;$$
$$v(K_1) = 0,70; v(K_2) = 0,75; v(K_3) = 0,80;$$
$$v(K_4) = 0,85; v(K_5) = 0,90; v(K_6) = 0,95; v(K_7) = 1; v(K) = 5,95.$$

Thus, the regional authorities choose $v^*(K_7) = (1,\ 1,\ 1,\ 1,\ 1) = 1$ and the corresponding budget $(11 \le R_1 \le 21,\ 14 \le R_2 \le 21,\ 19 \le R_3 \le 26,\ 18 \le R_4 \le 26, 18 \le R_5 \le 25)$.

$$v^*(K) = \sup\left\{ \sum_{K \in I} \mu_K v(K) / \sum_{j=1}^{n} b_j^{\min} \le \sum_{i=1}^{m} R_j^i \le \sum_{j=1}^{n} b_j^{\max} \right\} = 5,95 / 80 \le R_7^5 \le 119.$$

The Shapley vector for the given game is as follows:

$$v(I_1) = v(I_2) = \eta(i=3)[(0,70-0) + (0,75-0) + (0,80-0)] + \eta(i=4)[(0,85-0) +$$
$$+(0,90-0) + (0,95-0)] + \eta(i=5)[(1-0)] = \eta(3)[2,25] + \eta(4)[2,7] + \eta(5)[1] = 0,68;$$
$$v(I_3) = \eta(3)[(0,70-0)] + \eta(4)[(0,85-0,80) + (0,90-0,75) + \eta(5)[(1-0,95)] =$$
$$= \eta(3)[0,70] + \eta(4)[0,2] + \eta(5)[0,05] = 0,1359;$$
$$v(I_4) = \eta(3)[(0,80-0)] + \eta(4)[(0,85-0,70) + (0,95-0,75) + \eta(5)[(1-0,90)] =$$
$$= \eta(3)[0,80] + \eta(4)[0,35] + \eta(5)[0,1] = 0,1674;$$
$$v(I_5) = \eta(3)[(0,75-0)] + \eta(4)[(0,90-0,70) + (0,95-0,80) + \eta(5)[(1-0,85)] =$$
$$\eta(3)[0,75] + \eta(4)[0,35] + \eta(5)[0,25] = 0,1667.$$

Note that we do not expect the exact equality of coordinate sum of Shapley vector to one. It is enough that all conditions of the division be made with accurate calculations.

Besides, the calculated coordinates of the fuzzy Shapley vector should be considered only as a possible assessment or recommendation.

Assuming that in coalition K_1 players have varying degrees of involvement, for example $\mu_{K_1}(I_1) = \mu_{K_1}(I_2) = 1$, $\mu_{K_1}(I_3) = 0,7$, then the relationship between players can be represented by a fuzzy graph shown in Figure 1.

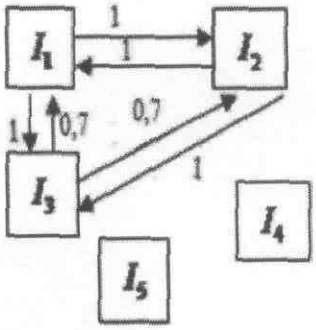

Fig. 1. *The fuzzy graph for players' relationships*
(source: authors' own example)

Therefore, players' winning in each coalition depending on the degree of participation will be:

$$v_{K_j}(i) = \mu_{K_j}(i) * v(K_j) / \sum \mu_{K_j}(i), \text{ i e. } v_{K_1}(I_1) = v_{K_1}(I_2) \approx 0,26, \ v_{K_1}(I_3) \approx 0,18.$$

Thus, using the apparatus of fuzzy coalition games, an approach can be implemented to organize cooperation and distribution of winning for participants in the regional innovation system, who, in order to achieve maximum efficiency in the performance of regional innovation programs, can form coalitions.

Despite the fact that participants may decide, for example, on the task of winnings distribution by "concepts", the model can show rational boundaries of distribution. Additionally, when using the budget money, regional authorities can influence this process through the model. Therefore, it can solve a number of tasks which are related to the payoff function, superadditivity, the fair distribution of maximum winning.

4. Modelling of participants motives when the formation of fuzzy coalitions

At the regional level, one of the most important functions of regional administration as a mediator is to ensure the cooperation of all stakeholders. The forming of coalitions of innovative participants in the region is a dynamic process, where regional administration solutions aim at achieving certain goals.

It should be noted that, as a participant in coalitions, administration may consider not only the residents, because (under certain conditions) external potential in the form of technological, organizational or market information may be critically needed.

Accordingly, two types of coalitions can be distinguished: local and transterritorial, the latter expanding the circle of ongoing projects thanks to resources of exogenous participants.

Furthermore, it is an open system, since it is affected by external factors. Here a bilateral interaction effect occurs, because by creating new knowledge and technologies, the innovation system has an impact not only on its environment but also on the external one. Therefore, the formation of coalitions is an open process, which can be characterized by positive or negative externalities.

As already remarked, it is a multi-stage process of formation and implementation of the goals for a coalition of innovative region participants, a process which develops over time as a logical sequence of decision making by regional administration, which aims to achieve certain goals in conditions of existing restrictions.

Such decisions are taken in a long time, when earlier decisions may affect the implementation and effectiveness of the subsequent stages. Since under such conditions the regional administration, as well as potential innovative participants, have a lack of clear understanding of the cooperation process, such coalitions can be characterized as fuzzy (uncertain).

Suppose that for performing innovation programs $IP = \{IP_h, h = \overline{1,H}\}$, within which it is necessary to complete several projects $\Pi = (\Pi_l, l = \overline{1,L})$, from a plurality of potential participants with different competencies $P = \{P_i^{(w)}, i = \overline{1,I}\}$ and resource capabilities the region needs to form coalitions $K_n, n = \overline{1,N}$.

Then we have the dynamic fuzzy coalitional game, which is determined by $(P, X, \tilde{V})$, where $P = \{P_i^{(w)}, i = \overline{1,I}\}$ a finite and non-empty set of innovative players, $X = \{x_t\}$ state of the system at the moment t, where $x_t, x_{t+1} \in X$ - a set of states of coalitions formation at the time point $t \in [0,T]$, $\tilde{V}$ — payoff function.

Whereas $\tilde{V}: 2^I \to \Re^+$ connects the coalition $K \subseteq P$ with the fuzzy winning value $\tilde{V}(K)$, then the fuzzy coalition K on the set of players P is characterized by the membership function $\mu_K : R \to [0,1]$, $K \neq 0$, $Sup\mu_K(\tilde{V}(K)) = 1$.

And besides, two groups of participants can be distinguished in the framework of the fuzzy dynamic cooperative game:

– **the first:** potential or individual participants $P_i^w, i=\overline{1,I}$, each of which is characterized by the following set (or ordered cortege)

$$\prec g_i, r_i, s_i, e(\tilde{v}(r_i)) \succ,$$

where g_i – target state of the system for the player i;

r_i – a resource that has a player i, $0 < r_i \le r_i^+$;

s_i – set of strategies of player i;

$e(\tilde{v}(r_i))$ - efficiency of player i in material form;

– **the second**: coalition or team of participants $K_n, n=\overline{1,N}$, where K is a non-empty subset of P – $K \subseteq P$.

As for certain dynamical systems, the process of forming a coalition can be described by a transition state equation, i.e. a dynamic model of coalition formation can be viewed as a set of objects of a kind:

$$\Gamma = \prec P, W, X, G_n, R_n, S_n, E(V(R_n)) \succ,$$

where: $P=\{P_i, i=\overline{1,I}\}$ a finite and non-empty set of innovative players in the region;

$W=\{w_i\}$ – competence of player i, because when performing innovative projects the importance (competence) of players is not equal;

$X=\{x_t\}$ – state of the system at moment t.

G_n – target state of the system for the coalition K_n;

R_n – resource now possessed by coalition K_n;

S_n – a lot of strategy for the coalition K_n;

$E(\tilde{V}(R_n))$ – the efficiency of the coalition K_n in material form.

Each potential participant will take part in a coalition K_n with sharing $s=(s_1+s_2+...+s_i)$ and winning $\tilde{V}(K_n)$, if the solution for command or the cooperative game will satisfy budget constraints (2) and the division vector takes place (3).

The combination of potential innovative players in coalitions K_n is possible if for players $a, d, ..., z \in P$ the following inequality holds:

$$E(\tilde{V}(R_n)) \ge e(\tilde{v}(r_a)), e(\tilde{v}(r_d)), ..., e(\tilde{v}(r_z)).$$

At that level, regional authorities can make a selection of potential innovative participants such that the total quantity $r_i^w, r_{Ip}^{i(w)}$ satisfies the requirements of the program execution. In other words, it can make the ordering of participants in accordance $r_i^w \leftrightarrow r_{Ip}^{i(w)}$, so that $\sum_{i=1}^{I}\left(r_i^w - r_{Ip}^{i,w}\right) \rightarrow \min$,

where r_i^w – resources at the disposal of the participant i with competencies w;

$r_{Ip}^{i(w)}$ – resource budget of participant i c with competencies w for the innovation program.

If the winning of the participant i from a resource investment r_i is $v(r_i, R)^{(i)}$, it is necessary to select that amount for which $\sum_{i=1}^{m} V\left(r_i, R\right)^{(i)} \rightarrow \max$.

Acting as a mediator between innovative players, a regional authority may recommend participants in the innovation process, as an optimal coalition with the corresponding budget, that for which $\sum_{K \in I} \mu_K v(K)$ is maximal. Accordingly, the regional authority selects $v^*\left(K_j\right)$ relevant budget according to (1) and (4).

As already noted, the open process of coalition formation is characterized by positive or negative externalities, from which it follows that the formation of the coalition will be effective, if the sum of the benefits and of the external effects will exceed the price of the program.

The coalition strategy $S_n \in [0,1]$ influenced the innovative strategies participants' map, which the participant can accept (1), reject (0) or participate in to a certain extent.

The relationship $l(a,b,...,z)$ between players $a,b,...,z \in P$ is located in the interval [0,1]. The value of the relationships between players a,b is $E_{a|b} = \frac{E_{a-b} + E_{b-a}}{2}$, while for the players n it is equal to $E_{a|z} = \frac{\sum_{i=1}^{n}\left(E_{a|z}\right)_i}{n}$.

At every moment t, the coalition K can be located in the state x_t, therefore, the state of the coalition can currently be described as follows: $f_{K_n}(x_t)$, and all states - $f_K(X) = \cup f_{K_n}(x_t)$.

If the characteristic function of the cooperative game or payoff function $\tilde{V}(K_n)$ also determined on the set $X = \{x_t\}$ – $\tilde{v}(i): X \rightarrow \Re$, and d_i is discounting, then winning for the player i during period $t = \overline{0,T}$ will be $\sum_{t=0}^{T} d_i^t \tilde{v}(i \mid x_t)$.

As in (5), the equitable redistribution of the maximum fuzzy winning performed by the Shapley vector, but already for $\tau \in [t_0, T]$, has the following form:

$$\varphi_{Sh}^{i}(v(i)^{\tau}) = \sum \frac{(m-k-1)!k!}{m!} \times [(\tilde{V}(K^{\tau}(\tau, X_K^{\tau})) - \tilde{V}(K / i^{\tau}(\tau, X_{K/i}^{\tau}))],$$

where $[(\tilde{V}(K^{\tau}(\tau, x_K^{\tau})) - \tilde{V}(K / i^{\tau}(\tau, x_{K/i}^{\tau}))]$ - limiting the contribution of player i into a coalition K at time point τ ;

X_K^{τ} - coalitions state K at time point τ .

5. Numerical example for approach of participants motives modelling

Suppose that for the implementation of the innovative program, consisting of 3 projects with an overall budget $R = 145.2$, it is necessary (at regional level) to create a team of performers (from 10 participants of innovation) consisting of 6 members. But the region, as well as the participants face the choice to take part fully, partially or refuse.

Within such a dynamic fuzzy cooperative game, there are various groups of participants: prospective or individual participants: financial institutions, businesses, scientific - research centres, government agencies etc., wherein each participant $P_i^w, i = \overline{1,10}$ is characterized by a certain set of target system status, available resources, a plurality of strategies and efficiency in material form.

Of course, they have many different goals:

- for financial institutions - to increase profitability by increasing the customer base, improving their image and reliability, strategies: business partnership and regional development;

- for companies - to increase profit, reliability, competitiveness, market share, to give an opportunity of self-accomplishment for performers of innovations etc., strategies: offensive to increase the competitiveness, imitative to retain market and technological advantages;

- for scientific research centres - to develop scientific potential, professional adaptation, commercialization of research projects, knowledge transfer etc.

In addition, objectives and strategies of regional authorities and governmental institutions can be noticed; as mediators, they are permanent participants:

- for the regional administration, one of the goals is to ensure the socio-economic development of the region, strategies: focusing, providing the status of the high-tech developed region;

- for government institutions, goals are an extension of knowledge reproduction, ensuring an effective innovation system, strategies for improving competitiveness at the international level etc.

Experts give linguistic estimations on objectives, strategies and resources for potential participants and coalitions in general, respectively, $L^{(i)}{}_G, L^{(i)}{}_S, L^{(i)}{}_r$, $L^{(K)}{}_G, L^{(K)}{}_S, L^{(K)}{}_r$.

The participant i will take part in the coalition if:

- his objectives, strategies and resource capabilities completely coincide with the objectives, strategies and resource requirements of the coalition and gives additional opportunities $L^{(i)}{}_{G,S,r} \subset L^{(K)}{}_{G,S,r}$, for example, gives the company the opportunity to profit, improve competitiveness, to increase market share, entering the international market, fiscal privileges; then the degree of participation is $\mu_{K_n}(i) = 1$;

- partially match up with the coalition goals, for financial organization getting profit, ensuring partnership and regional development: $0 \prec \mu_{K_n}(i) \prec 1$.

- do not match: $\mu_{K_n}(i) = 0$.

Having designated a fuzzy value of resources available to players I_i and transferring to other players of coalitions K, as $R_i^{K_j} = (r_i^L, r_i^*, r_i^R)$ – fuzzy LR – number, we can build a fuzzy table of resource allocation (Table 2).

$R_i^{K_j}$	I_1	I_2	I_3	I_4	I_5
R_1	(3, 3.5, 4.5)	(2, 2.5, 3)	(6,7, 8)	(2, 2.5,3)	(3, 4, 5)
R_2	(2.5, 3.5, 4)	(5, 6, 7)	(3,3.5, 4)	(4,5,6)	(4, 5, 5.5)
R_3	(2, 2.5, 3)	(1, 1.5, 2)	(5, 5.5, 6)	(1, 2,3)	(6,7,8)
R_4	(1, 1.5, 2.5)	(1,1.5, 2)	(3,3.5, 4)	(6, 7, 8)	(5, 6,7)
R_5	(3.5, 5, 6)	(3, 3.5, 4)	(1, 1.5, 2)	(2, 3.5, 4)	(2, 3, 4)
$\sum \oplus$	(12, 16, 20)	(12, 15, 18)	(17, 21, 24)	(15, 20,24)	(20, 25, 29.5)
$R_i^{K_j}$	I_6	I_7	I_8	I_9	I_{10}

R_6	(2, 2.5, 3)	(8, 9, 10)	(2, 4, 6)	(2, 2.5, 3)	(3, 4, 4.5)
R_7	(1, 2, 3)	(6, 7, 8)	(2, 3, 5)	(2, 3, 4)	(4.5, 5, 5.5)
R_8	(6,7,8)	(1, 1.5, 2)	(6, 6.5, 7)	(4, 5, 5.5)	(2, 2.5, 3)
R_9	(3, 3.5,4)	(1,2,3)	(4,4.5, 5)	(6, 6.5, 7.5)	(4, 5, 5.5)
R_{10}	(4,5,6)	(1,2.5,3)	(1, 2, 3)	(5,6,7)	(1.5, 2.5, 3.5)
$\sum\oplus$	(16, 20, 24)	(15, 21, 26)	(15, 20, 26)	(19, 23, 27)	(15, 19, 21)

Table 2. *Allocation of resources*
(source: authors' own numerical example)

Without the participation of I_5, I_9, the winning of the coalition is equal to $v(K)=0$; thus in any circumstances, they are binding participants of the coalition. Each participant has the following individual usefulness $\tilde{v}(1)=3$, $\tilde{v}(2)$=3.2, $\tilde{v}(3)=3.5$, $\tilde{v}(4)=\tilde{v}(6)=\tilde{v}(8)=4$, $\tilde{v}(5)=8$, $\tilde{v}(7)=5$, $\tilde{v}(9)=6$, $\tilde{v}(10)=5.5$, and $v(K)=74$.

Therefore:

$$s=(s_1+s_2+...+s_i)=46.2\le\tilde{V}(K_n)=74,\ \sum_{i\in K_n}\alpha_i=\tilde{V}(K_n)-\sum_{i\in K_n}\tilde{v}(i)=27.8$$

Despite the fact that the division and effectiveness of the coalition in material form satisfy all potential participants, I_2, I_{10} choose the "red" path: $\mu_K(2)=\mu_K(2)=0$, because their goals, strategies do not coincide with the coalitional ones.

The regional authorities perform a selection of potential innovative participants so that the total value of their resources satisfies the requirements of the program.

Participants I_7, I_8 have equal competence, but $\mu_K(7)=0.6$, $\mu_K(8)=0,9$ from which it follows that player I_7 will not fully realize its potential, i.e. administration gives "red light" for I_7 and "green" for I_8.

But for participants I_4, I_6 who also have the same competences and approximately equal resource potentials, but $\mu_K(4)=0.8$, $\mu_K(6)=0.7$, I_4 will get "green colour" as the main participant, and I_6 will be involved in the implementation under certain conditions. Then $K=(1,0,1,1,1,0.5,0,1,1,0)$, and

$$v^*(K) = \sup\left\{\sum_{K\in I}\mu_K v(K) / \sum_{j=1}^{n} b_j^{\min} \le \sum_{i=1}^{m} R_j^i \le \sum_{j=1}^{n} b_j^{\max}\right\} = 74/114 \le R_7^5 \le 174.5\,.$$

The dynamic fuzzy cooperative game redistribution of maximum fuzzy winnings at time moment $\tau \in [t_0, T]$ can be performed by the Shapley vector.

In this way, based on the fuzzy dynamic model, coalitions of innovative participants in the region can be configured, when, under certain abstract conditions, the regional administration, as well as potential participants, make decisions in accordance with their interests.

6. Model of interaction "state - regions" as a hierarchical fuzzy game

As already mentioned, regions and their interaction play a certain role in the production of innovations. At the same time, they largely depend on the national context. And such "solidarity" politics follows from the "status" and, therefore, opportunities for regions.

Because the state tries to lead an innovative system in such a state, which will be beneficial to all, then, under conditions of uncertainty, it makes decisions, relying on a fuzzy understanding of possibilities and strategies of regions, which may be related to the participants' goals, time of process completion etc.

At abstracted conditions, there is a hierarchy of subjects, and consequently, a fixed sequence of actions (moves, steps). I.e. in order to implement its innovation policy, the state sets the rules of the game, whereupon it acts accordingly, on already established regions. The model of this form of interaction between the state and regions can be represented as a hierarchical fuzzy game.

We have a game $(S|R,\widetilde{G}|\widetilde{g}_i)$, where $S\,|\,R$ players: S – the state and R – regions $R_i \in R, i = 1,\dots,I$, $\widetilde{G}\,|\,\widetilde{g}_i$ – the fuzzy goal function or criteria of effectiveness of the state and regions.

Participants have fuzzy criteria of effectiveness of, accordingly, $\widetilde{G} = F(X_j, x_i)$ and $\widetilde{g}_i = f(X_j, x_i)$, which are dependent on government actions $X_j \in X, j = \overline{1,J}$ (it may be laws, regulations, incentives, penalties and, eventually, financing of innovative activity in regions) and regions $x_i \in x, i = 1,\dots,I$.

If it is assumed that the state chooses its innovation policy and announced this to regions, then for regions it is only necessary to choose (from a variety of actions) some, at which the goal function of the region will be maximal:

$$\widetilde{\Phi}(X) = Sup(f(X_j, x_i)) \tag{6}$$

It is obvious that (6) depends on $X_i \in X$, and the assessment of results for the state $\widetilde{G} = F(X_j, x_i)$, which is also due to the actions of regions, can be optimistic or pessimistic. So, if the choice of region is favourable, $X^* \in \arg\max\max F(X_j, x_i)$, in case of unfavourable $\widetilde{\Phi}(X)$ the state chooses a minimum gain, which is already maximized by its actions: $X^* \in \arg\max\min F(X_j, x_i)$.

As known, the state (whose budget is limited) is one of the financial sources for regional innovation programs.

Let us assume that the government funding depends on: firstly, actions or games of regions; secondly, fuzzy characteristics, which a region has $\Gamma = \prec V_F, L_{ID}, \psi \succ$, where V_F – the sum of funding я, L_{ID} – level of innovation development, ψ – innovative "productivity".

Depending on the activity of the region, the benefit distribution function can be $\mu_\psi(\widetilde{g}_i) \in [0,1]$, and the sum of financing i will depend on $\mu_\Gamma(R_i) \in [0,1]$ – number characterizing the level of assessment.

Then the winnings of state will be: $S = \psi - V_F$, and for the region: $\widetilde{g}_i(V_F, L_{ID}, \psi) = V_F - c(V_F, \psi)$.

If we denote the degree of state certainty at the feasibility of investments in the region i through $\mu_S(i)$, then its winnings will be: $\mu_S(i)\psi$, i.e. the choice of the state depends on actions of the region $X = \widetilde{X}_i(x_i)$.

In case the region acts on the state "plan" ($p \in P$), then it is encouraged, otherwise, it waits for "penalty", where its degree is in inverse dependence on a number of "trespassers":

$$\widetilde{X}_i(x_i) = \begin{cases} encouragement, if \quad x_i \in P \\ penalty, if \quad x_i \notin P \end{cases}.$$

State strategy of penalties $X_i = \widetilde{X}_i(x_i)$ determined from condition:

$$f_i(\widetilde{X}_i(x_i), x_i) = \min f_i(X_i, x_i).$$

The criterion of the effectiveness of the state – it is the difference between revenue and the sum of financing: $V(\widetilde{X}_i(x_i), x_i) = H(x_i) - \sum_{i \in I} \widetilde{X}_i(x_i)$.

The goal function of the region will be: $f_i(\widetilde{X}_i(x_i), x_i) = \widetilde{X}_i(x_i) - c_i(x_i), i \in I$.

Hence, for an optimal plan, the state has to seek $H(x) - \sum_{i \in I} c_i(x_i) \to \max$.

Proceeding from the fact that the state funds are limited, regions are becoming weakly bound: $\sum_{i \in I} \widetilde{X}_i(x_i) \le B$ *or* $\sum_{i \in I} c_i(x_i) \le B$, i.e. the optimization task can be represented in the following manner: $H(x) - \sum_{i \in I} c_i(x_i) \to \max$, $\sum_{i \in I} c_i(x_i) \le B \quad \left| R_i \in R \right.$.

Since the budget constraint binds all regions, then both costs and winnings of each region depend on the actions of others. Then, the solution of the problem will be the Nash equilibrium:

$$E_{Nash} = \left\{ x_i^I \in x \left| \widetilde{X}_i\left(x_i^I\right) - c_i\left(x_i^I\right) \ge \widetilde{X}_i\left(x_i^I, x_{-i}^I\right) - c_i\left(x_i^I, x_{-i}^I\right), i \in I, x_i \in x \right. \right\}.$$

7. Numerical example for modelling of "state - regions" interaction

Suppose that there are expert assessments criteria of effectiveness both for the region and for the state. Strategies of state and region are $X_i, x_j, i = \overline{1,4}, j = \overline{1,4}$, which are given by the following matrix:

$$M = \begin{bmatrix} X_i / x_j & x_1 & x_2 & x_3 & x_4 \\ X_1 & \upsilon[H,M] & \upsilon[M,H] & \upsilon[VL,L] & \upsilon[H,M] \\ X_2 & \upsilon[M,H] & \upsilon[L,M] & \upsilon[M,L] & \upsilon[VH,H] \\ X_3 & \upsilon[VH,M] & \upsilon[H,VH] & \upsilon[VH,M] & \upsilon[M,L] \\ X_4 & \upsilon[L,VL] & \upsilon[VH,H] & \upsilon[M,H] & \upsilon[VL,L] \end{bmatrix}, \quad \begin{matrix} VL \\ L \\ M \\ H \\ VH \end{matrix} \begin{bmatrix} 0 & 0.2 & 0.4 \\ 0.35 & 0.4 & 0.5 \\ 0.45 & 0.55 & 0.65 \\ 0.6 & 0.7 & 0.8 \\ 0.75 & 0.85 & 1 \end{bmatrix},$$

where *VL,L,M,H,VH* are linguistic estimates, correspondingly, very low, low, medium, high, very high.

The characteristic of the region is given by the following fuzzy linguistic assessments:

- required sum of financing < minimum, average, maximum > ≡ $V_F = [M^-, M, M^+]$;

– level of innovation development of region = < unsatisfactory, satisfactory, high >≡ $L_{ID} = [S^-, S, S^+]$;

– innovative "productivity" = < low, medium, high >≡ $\psi = [P^-, P, P^+]$.

$$V_F = \begin{matrix} M^- \\ M \\ M^+ \end{matrix}\begin{bmatrix} 0.3 & 0.4 & 0.6 \\ 0.5 & 0.65 & 0.75 \\ 0.7 & 0.85 & 1 \end{bmatrix}; L_{ID} = \begin{matrix} S^- \\ S \\ S^+ \end{matrix}\begin{bmatrix} 0.1 & 0.3 & 0.5 \\ 0.4 & 0.6 & 0.8 \\ 0.75 & 0.9 & 1 \end{bmatrix}; \psi = \begin{matrix} P^- \\ P \\ P^+ \end{matrix}\begin{bmatrix} 0.2 & 0.35 & 0.45 \\ 0.4 & 0.55 & 0.75 \\ 0.7 & 0.8 & 1 \end{bmatrix}.$$

At first, the state (from the matrix M) chooses the strategy, where the criterion of effectiveness is maximal $X_{ij} = X_{31} = \upsilon[VH, M]$, and the region is average. Then, it is possible to formulate the following base of fuzzy rules:

$R^{(1)}$, if the region operates only in its own interests, the choice is held on the principle of maximum;

$R^{(2)}$: if the region takes into account the interests of all, then the choice runs along the minimax principle.

Secondly, on the basis of the state strategy, the region could proceed as follows:

- it can (on the basis of its features) act only in its own interests, thereby selecting $x_{ji} = x_{23} = \upsilon[H, VH]$;

- or, for the creation of a favourable environment, it can choose $x_{13} = x_{33} = \upsilon[VH, M]$, when winnings for the state is maximal, and for the region it is average. In the case the region acts only in its own interests, the state will win not less than 0.7.

For the characteristic of the region we have the following summarizing experts' assessments: $V_F = M^+, L_{ID} = S, \psi = P$. Then the number characterizing the level of assessment, on which the sum of financing will depend $\mu_\Gamma(R) \equiv V_F \wedge L_{ID} \wedge \psi \in [0.7 - 0.75]$. If the effect provided by innovative productivity is δ, then the state winning is $\delta - V_F$, and the region winning is $\delta - c(V_F, \psi)$.

In case the region receives funding for the implementation of innovative projects in frameworks of specific programs, but does not use it for its intended purpose, then the state may apply methods of "penalty".

For example, if the state finances an innovative project and, at some stage, detects failure to comply with the plan, the state can identify causes of this, stop funding and determine "penalties": fines etc.

Suppose that the state, in addition to the annual financing of the regions, provides additional funds ($\Phi = 8$), based on their requests for funding. There are 3 regions ($R_i, i = \overline{1,3}$), whose requests for monetary funds $q_i, i = \overline{1,3}$.

In case $\sum q_i \leq \Phi, i = \overline{1,3}$, financing is carried out according to demands, otherwise, if $\sum q_i \succ 8, i = \overline{1,3}$, it takes into account characteristics of regions, for which there are the following estimates: $\mu_\Gamma(R_1) = 0{,}5$; $\mu_\Gamma(R_2) = 0{,}7$; $\mu_\Gamma(R_3) = 0{,}8$.

Then, taking into account existing characteristics, the financing values for regions can be determined by the following formula:

$$\Phi(R_i) = \frac{\mu_\Gamma(R_i) * \Phi}{\sum\limits_{i=\overline{1,3}} \mu_\Gamma(R_i)}, \text{ i.e. } \Phi(R_1) = 2, \quad \Phi(R_2) = 2{,}8, \quad \Phi(R_3) = 3{,}2.$$

Accordingly, each region within the framework of such an approach will ensure a high level of $\mu_\Gamma(R_i)$ for further development.

Because state financial resources are limited, then these 3 regions become weakly coupled and the best plan for the state is to define an optimization problem of linear programming:

$$\widetilde{V}_S \to \max, \ \sum_{i \in I} c_i(x_i) \leq B \quad \middle| R_i \in R,$$

where $\widetilde{V}_S$ - the fuzzy goal function for the criterion of state efficiency, defined as $\widetilde{V}_S = H(x) - \sum\limits_{i=\overline{1,3}} c_i(x_i), H(x) \geq 0, c_i(x_i) \geq 0$; $H(x)$ and $c_i(x_i)$ - accordingly, functions of state income and expenses of the region i. The criterion of effectiveness is set by the intervals $[\underline{V_S}, V_S, \overline{V_S}]$ (low, middle and high, Figure 2.

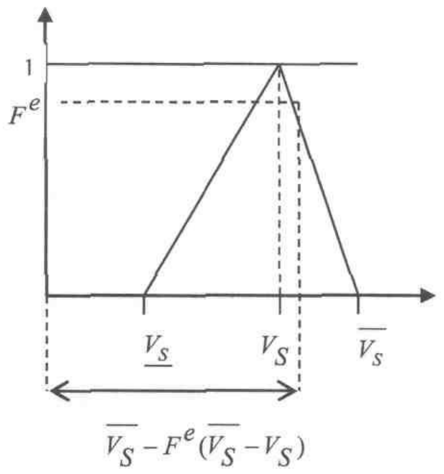

Fig. 2. *Membership functions of linguistic evaluations*
(source: authors' own example)

The goal function will have the following form: $\max(\overline{V_S} - F^e(\overline{V_S} - V_S))$ (7),

where F^e - degree of experts confidence; $\widetilde{V}_S = H(x) - \sum_{i=\overline{1,3}} c_i(x_i)$,

$H(X) = x1 \equiv [8,9,11]$, $c(x) = x2 \equiv [7.5,8,9]$.

Then $f(\underline{V_S}, V_S, \overline{V_S}) = [8H(X) - 7.5c(x), 9H(X) - 8c(x), 11H(X) - 9c(x)$.

Because the goal function is (7), and $F^e = 0.9$, then

$$F^{\max} = 10.1 * x1 - 8.1 * x2,$$

$$\begin{cases} 8 * x1 - 3 * x2 \le 50 \\ 9 * x1 - 7.5 * x2 \ge 28. \\ x1 \ge 0, x2 \ge 8 \end{cases}$$

It is possible to build the area of feasible solutions, i.e. graphically solve a system of inequalities as shown in Figure 3. To do this, every straight line should be constructed and the half-plane should be defined by the corresponding inequalities.

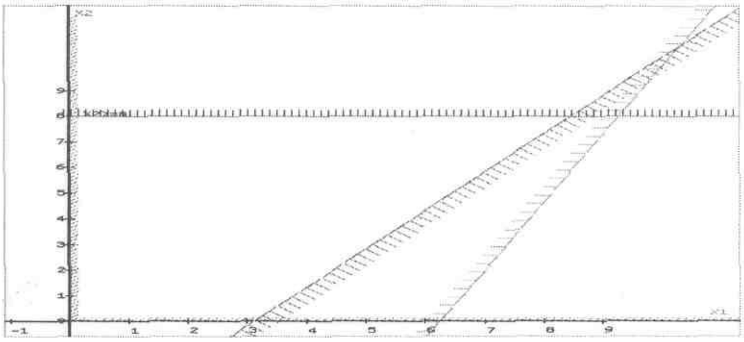

Fig. 3. *Graphical solution*
(source: own authors' example)

Solving the system of equations, we get: $x1$ = 9.25, $x2$ = 8. Thus, the sought maximum value of the goal function will be: $F(X)$ = 10.1*9.25 – 8.1*8 = 28.6.

8. Conclusions

The paper considered the possibilities of economic and mathematical modelling of the interaction processes of various participants in the innovative development of regions with the use of game theory approaches in the fuzzy-multiple setting. The possibility and justification of the feasibility of such a simulation are shown.

The tasks of the interaction of innovative participants in the region in the form of fuzzy coalition games are also successively considered. The possibility of taking into account the reasons of the participants in the formation of the corresponding fuzzy coalitions, the modelling of the interaction "state-regions", presented as a fuzzy hierarchical game, are shown. Appropriate examples of numerical calculations are given.

The proposed models have practical value and utility. They can be successfully used to improve the interaction of the parties in the process of implementing various innovative projects and activities at various levels of economy and management.

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